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Acceptance sampling plan of accelerated life testing for lognormal distribution under time-censoring

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Abstract Lognormal distribution is commonly used in engineering. It is also a life distribution of important research values. For long-life products follow this distribution, it is necessary to apply accelerated testing techniques to product demonstration. This paper describes the development of accelerated life testing sampling plans (ALSPs) for lognormal distribution under time-censoring conditions. ALSPs take both producer and consumer risks into account, and they can be designed to work whether acceleration factor (AF) is known or unknown. When AF is known, life testing is assumed to be conducted under accelerated conditions with time-censoring. The producer and consumer risks are satisfied, and the size of test sample and the size of acceptance number are optimized. Then sensitivity analyses are conducted. When AF is unknown, two or more predetermined levels of accelerated stress are used. The sample sizes and sample proportion allocated to each stress level are optimized. The acceptance constant that satisfies producer and consumer risk is obtained by minimizing the generalized asymptotic variance of the test statistics. Finally, the properties of the two ALSPs (one for known-AF conditions and one for unknown-AF conditions) are investigated to show that the proposed method is correct and usable through numerical examples.

1. Introduction

Life testing sampling plans (LSPs) are usually used to determine whether to accept or reject batches of products when lifespan is an important index. There are many studies on the design of LSPs. The differences among them are mainly in the assumed lifetime distribution (exponential and Weibull lifetime distributions), censoring scheme (time-censoring or failure-censoring) and testing conditions (accelerated or use conditions). As science and technology have improved, product reliability has increased and product life has been extended. This makes it difficult for traditional reliability and life demonstration testing to judge product indexes. For example, in the traditional reliability demonstration testing scheme under exponential distribution, when the producer risk and consumer risk are 20%, the testing time is 4.3 times longer than the MTBF inspection limit. That means if the MTBF is 2000 h, the testing time is 8600 h. This is unacceptably long.

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Accelerated life testing (ALT) is used to facilitate fast acceptance. This shortens testing time and reduces development costs through the use of harsher-than-use testing conditions.

There are many studies on the design of LSPs that involve exponential lifetime distribution. In 1980, Spurrier and Wei[1] developed Type-I censoring and produced LSPs that consider only the producer's risk. In 1995, Jeong and Yum[2] expanded this design to cases in which both types of risk are considered. Kim and Tum[3] presented another LSPs design involving Type-I censoring and intermittent monitoring in 2010. Muhammad Aslam et al.[4] published a study on acceptance sampling plans for generalized exponential distribution when the lifetime experiment is terminated at a pre-determined time. For Weibull distribution, Fertig and Mann[5] discussed sampling plans for use with Weibull distribution and constructed the hybrid censored LSPs. In 2000, Balasooriya et al.[6] first presented progressively failure-censored LSPs. In 2004, Balasooriya and Low[7] expanded upon the results of the previous Balasooriya[6] study to cases involving competing causes of failure. In 2004, Chen et al.[8] presented Bayesian LSPs under hybrid censoring conditions with prior information on the shape and scale parameters of the Weibull lifetime distribution. In 2009, Aslam and Jun[9] produced a group acceptance LSP for a truncated life testing. This LSP was able to test multiple items simultaneously. In 2013, Ismail[10] designed a step-stress accelerated life test under failure-censoring conditions assuming the Weibull distribution with Type-II censored data.

In engineering, lognormal distribution plays an important role in statistically predicting the fatigue life of mechanical products. In 1962, Gupta[11] studied life testing sampling plans for truncated life tests from the normal and lognormal distributions. In 1989, Schneider[12] discussed the design of variable-sampling plans based on failure-censored samples. This method of design can be applied to lognormal and Weibull-distributed lifetimes. In 2000, Balasooriya and Balakrishnan[13] presented sampling plans for lognormal distribution. These plans are based on progressively censored samples. Large-sample approximations of the best linear unbiased estimators of the location and scale parameters are used. In 2006, Wu and Lu[14] proposed a statistical method for working out reliability sampling plans with Type-I censored samples for items whose failure times have either normal or lognormal distributions. In 2009, Srivastava and Shukla[15] presented an optimum simple ramp accelerated life test with two different linearly increasing stresses for log–logistic distribution under Type-I censoring.

In accelerated testing, when the acceleration factor (AF) is unknown, for exponential distribution, Yum and Kim[16] developed a life sampling plan for failure-censoring at two stress levels. However, the calculations required for this plan are very complicated and the error rate is large. In 1994, Hsieh[17] expanded upon Yum and Kim's work such to minimize the total number of failures at the stress level. In 1993, Bai et al.[18] became the first to develop the LSP for use under failure-censoring at two stress levels above ordinary-use conditions. Then, in order to study time censoring test plans, they[19] extended the case under expected test time constraints. In 2009, Seo et al.[20] designed accelerated life testing sampling plans (ALSPs) for cases in which the shape parameter of Weibull distribution is non-constant. Kim and Yum[21] designed an ALSP under time-censoring conditions by assuming that the shape parameter of Weibull distribution was unknown. Then they developed ALSPs under hybrid censoring conditions by assuming that the shape parameter of Weibull distribution was known.[22]

In engineering practice, time censoring is the most common way to cut a test short. For long-life products that follow lognormal distribution, it is necessary to develop ALSPS under time-censoring conditions. In this way, this paper discusses the design of ALSPs for lognormal distribution under time-censoring conditions. Taking both producer and consumer risks into consideration. ALSPs can be designed whether AF is known or not.

In Section 2, we depict the design of an accelerated life testing sampling plan based on the lognormal distribution under time-censoring conditions when AF is known and when it is unknown. In Section 3, we apply these two sampling plans to case study and sensitivity analysis is also conducted. Finally, conclusions are presented in Section 4.

2. Accelerated life testing sampling plan under time-censoring

2.1. Assumptions

Assumption 1. The lifetime of products under any stress levels follows lognormal distribution, and the cumulative distribution function can be expressed as

$$F(t) = \int_0^t \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2} dx = \Phi\left(\frac{\ln t - \mu}{\sigma}\right) \tag{1}$$

where μ is the location parameter and σ the scale parameter.

Assumption 2. The location parameter μ is acceleration model and satisfies a linear function of stress s. It can be written as follows:

$$\mu = \gamma_0 + \gamma_1 \varphi(s) \tag{2}$$

where γ_0 and γ_1 are the unknown parameters. Function $\varphi(s)$ is the function of stress s and could be different representations when different accelerate stresses are used. For example, if temperature is chosen, $\varphi(s) = 1/s$; if electrical stress, $\varphi(s) = \ln s$ or $\varphi(s) = s$. In addition, the acceleration factor is $AF = \mu_U/\mu_A$. Here, subscript "U" represents "use conditions" and "A" stands for "accelerated conditions." μ_U and μ_A are the mean of log lifetime of lognormal distribution under use and accelerated conditions, respectively.

Assumption 3. Failure mechanism will not change with stress levels, that means, the scale parameter σ keeps constant at different stress levels.

2.2. Accelerated life testing sampling plan when AF is known

The key to using the accelerated testing technology in LSPs is dealing with the acceleration factor. This section first discusses how to design ALSP when AF is known.

Since AF has been confirmed in advance, the life of product under accelerated conditions can be directly determined based on use conditions. Then the reliability of product at the time of censoring could be determined under accelerated conditions by using the operating characteristic (OC) curve. We can solve the equation regarding sample size n and acceptance number c

which satisfy both risks. Then a suitable sampling test plan (n, c) under accelerated conditions can be found.

2.2.1. Testing and acceptance procedures

According to Assumption 1, ALT and acceptance procedures should be as follows:

(1) n test items are randomly selected from lots and tested under accelerated condition. AF is known.
(2) Failed test items will not be replaced with new ones.
(3) ALT is terminated at the preset censoring time τ_A.
(4) Suppose k failures occur during the test. If $k < c$, the lots are accepted. Otherwise, rejected.

2.2.2. Acceptance sampling plan of ALT under time censoring

Under accelerated conditions, the life of products follows log-normal distribution and the location parameter $\mu_A = \mu_U/\mathrm{AF}$. The cumulative distribution function of the lifetime under accelerated condition can be rewritten based on Eq. (1) as follows:

$$F_A(t_A) = \Phi\left(\frac{\ln t_A - \mu_A}{\sigma}\right) \tag{3}$$

Then, under accelerated condition, hypotheses could be given as follows:

$$\begin{cases} H_0 : \mu_A = \mu_{A0} (= \mu_{U0}/\mathrm{AF}) \\ H_1 : \mu_A = \mu_{A1} (= \mu_{U1}/\mathrm{AF}) \end{cases} \quad \mu_{A1} < \mu_{A0}$$

In this plan, the mean lifetime serves as the abscissa of the OC curve. Here, μ_{U0} is the acceptable characteristic level under use condition and μ_{U1} is the rejectable characteristic level under use condition. These two pre-specified points μ_{U0} and μ_{U1} in the abscissa of OC curve (see Fig. 1) are the upper and lower limits of inspection in the statistical testing plan. They can be determined by the agreement between producer and consumer. Suppose the mission time t_{UM} under use conditions is given and the reliability at this time is $R_U(t_{UM}) = 1 - F_U(t_{UM})$. This indicates the following relationship:

$$\mu_U = \ln t_{UM} - \sigma\Phi^{-1}(1 - R(t_{UM}))$$

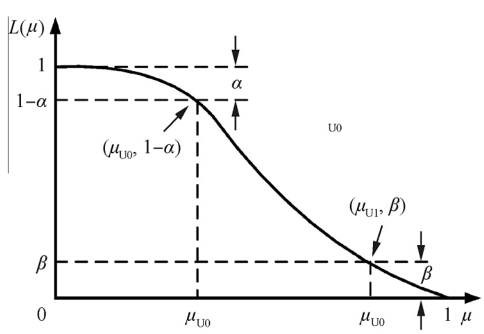

Fig. 1 Operating characteristic (OC) curve.

Accelerated life testing sampling plan satisfied the producer risk α and consumer risk β can be obtained by solving the equations with the sample size n and "rejection" number c as follows:

$$L(\mu_{A0}) = P(\text{accept} \ \ |\mu_A = \mu_{A0}) = \sum_{k=0}^{c-1} \binom{n}{k} (1 - q_0)^k q_0^{n-k} = 1 - \alpha \tag{4}$$

$$L(\mu_{A1}) = P(\text{accept} \ |\mu_A = \mu_{A1}) = \sum_{k=0}^{c-1} \binom{n}{k} (1 - q_1)^k q_1^{n-k} = \beta \tag{5}$$

where as indicated in Eq. (3), q_i ($i = 0, 1$) is the reliability of product at the censoring time τ_A under accelerated conditions. q_0 and q_1 are the acceptable and rejectable reliability level when $i = 0$ and $i = 1$, respectively.

$$q_i = 1 - \Phi\left(\frac{\ln \tau_A - \mu_{Ui}/\mathrm{AF}}{\sigma}\right) \tag{6}$$

where $\tau_A \times \mathrm{AF}$ is the equivalent "censoring time" under use condition. In this way, q_i can also be interpreted as the reliability of product that terminated at the censoring time τ_A under use conditions. In engineering, it is common to regard $\alpha = 0.2$ and $\beta = 0.2$ as normal risks. Therefore, suppose $\alpha = 0.2$ and $\beta = 0.2$, then different q_1 and q_0 and the corresponding n and c could be obtained by solving the above Eqs. (4) and (5). The results are shown in Table 1.

To explore how sampling plan changes with α and β, q_1 and q_0 are set as 0.88 and 0.98, respectively; α and β are changed to determine their influences on the sample size n and rejection number c, as shown in Fig. 1.

Using the sampling plans shown in Table 1 and Fig. 1, some of the properties of this plan could be summarized:

(1) For given α, β, and q_1, n decreases as q_0 increases.
(2) For given α, β, and q_0, n increases as q_1 increases.
(3) For given q_0 and q_1, n decreases as α and β increase.

Table 1 ALSPs $n(c)$ under time-censoring conditions when $\alpha = 0.2$ and $\beta = 0.2$.

q_1	q_0								
	0.99	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
0.98	453 (7)								
0.96	74 (2)	226 (7)							
0.94	49 (2)	71 (3)	337 (17)						
0.92	37 (2)	37 (2)	98 (6)	469 (33)					
0.90	29 (2)	29 (2)	54 (4)	135 (11)	587 (53)				
0.88	24 (2)	24 (2)	35 (3)	65 (6)	168 (17)	707 (78)			
0.86	21 (2)	21 (2)	30 (3)	47 (5)	80 (9)	190 (23)	806 (105)		
0.84	18 (2)	18 (2)	18 (2)	34 (4)	48 (6)	91 (12)	220 (31)	912 (137)	

(4) The "rejection" number c has properties similar to those of n.

2.3. Accelerated life testing sampling plan when AF is unknown

We may not be able to get enough data to estimate AF for most of the time in engineering practice due to the cost and time constraints, so ALSP with unknown AF is more practical than the one with known AF.

When AF is unknown, the lifetime of product cannot be found under the test only by one accelerated stress level. Two or more levels of accelerated stress should be designed and constant stress accelerated life testing (CSALT) could be used to collect time-to-failure data. In order to design the ALSP with unknown AF, test statistics were set up to verify and obtain the parameters of the test statistics using OC curves and both risks. The generalized asymptotic variance of the test statistics must be minimized to produce the test plan (see Fig. 2).

2.3.1. Assumptions and acceptance procedures

According to Assumptions 1 and 2, the procedures for ALSP with unknown AF are as follows:

(1) Without loss of generality, the life test uses two predetermined stress levels, s_1(low stress levels) and s_2 (high stress levels). Usually, s_2 should be less than endurable stress limit such that failure mechanism will keep the same as the normal condition. The exact value of s_2 can be approximated according to engineering experience. s_1 will be optimized.
(2) $n\rho$ test items are randomly selected from n samples and tested under stress s_1, and the rest are tested under stress s_2. The proportion ρ of items allocated to low stress is determined using an optimization method described in the next section.
(3) All test items are tested under the corresponding stress conditions. Failures are observed until the pre-specified censoring time η.

Suppose the lifetime of products which the lower limit is Z is assigned. Then the product that lifetime $T < Z$ is treated as a nonconforming product. In order to simplify the calculation process, the log lifetime $Y = \ln T$ was used instead of the actual lifetime T. Hence, Y follows a normal distribution with mean μ and variance σ.[2] The lower limit is $Z' = \ln Z$ and the acceptance procedures are as follows:

(1) n tested items are randomly selected from lots and tested in accordance with the above procedures.
(2) Evaluate (MLE) $\hat{\mu}_U$ and $\hat{\sigma}$ of parameters μ and σ under use conditions by using ALT data obtained.
(3) Compare the following test statistic X:

$$X = \hat{\mu}_U - k\hat{\sigma} \tag{7}$$

and Z'. If $X > Z'$, accept the lots; otherwise, reject.

In this plan, let p as the percent defective and the corresponding OC curve is shown in Fig. 3.

The sample size n and rejection constant c (method for determining see next section) must be determined so that lots with $p \leqslant p_\alpha$ are accepted with a high probability of at least $1-\alpha$ and lots with $p > p_\beta$ are accepted with a small probability of at most β. Here, α and β are producer and consumer risks respectively; p_α and p_β are the proportion of nonconforming products when the probabilities of acceptance are $1-\alpha$ and β through the OC curve.

2.3.2. Acceptance sampling plan of ALT under time-censoring

In this section, the sample size n, the low stress level s_1, the proportion ρ of products allocated to low stress and the acceptance constant k are optimized.

2.3.2.1. Stress standardization. To simplify the calculation process, the model is standardized. Standardized stress is defined $\xi = (s - s_0)/(s_2 - s_0)$. For use condition, $s = s_0$, $\xi = \xi_0 = 0$. For lower stress level, s_1, $\xi = \xi_1$ $(0 < \xi_1 < 1)$. For higher stress level, s_2, $\xi = \xi_2 = 1$. Substitute ξ into Eq. (2), we can get the following:

$$\mu = \lambda_0 + \lambda_1\xi \tag{8}$$

where $\lambda_0 = \gamma_0 + \gamma_1 \cdot s_2$, $\lambda_1 = \gamma_1(s_2 - s_0)$. Parameter $\mu_U = \lambda_0$ can be derived under use conditions.

2.3.2.2. Asymptotic variance of testing statistics. Because $\mu_U = \lambda_0$, the expression of test statistic Eq. (7) can be written as follows: $X = \hat{\mu}_0 - k\hat{\sigma} = \hat{\lambda}_0 - k\hat{\sigma}$. It is consistent with an asymptotically normal distribution for which expectation $E(X) = \lambda_0 - k\sigma$ and variance $\mathrm{Asvar}(X) = \mathrm{Asvar}(\hat{\lambda}$ $0) - 2k \cdot \mathrm{Ascov}(\hat{\lambda}_0, \hat{\sigma}) + k^2 \cdot \mathrm{Asvar}(\hat{\sigma})$. Here, $\mathrm{Asvar}(\hat{\lambda}_0)$,

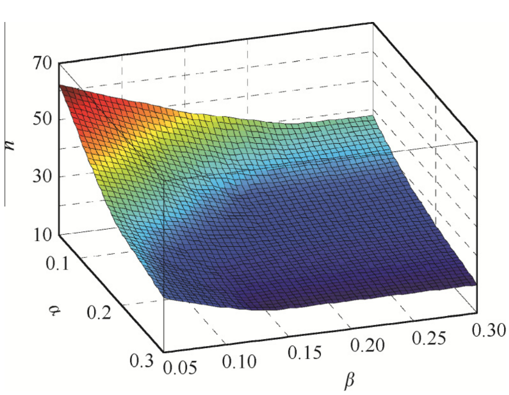

Fig. 2 Sample size change with α and β (when AF is known).

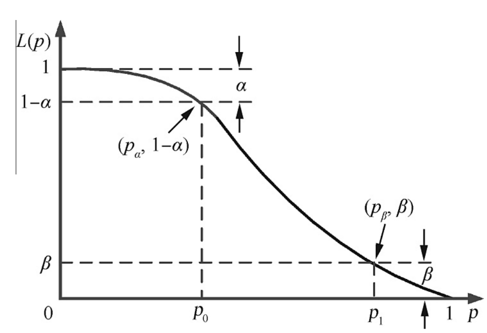

Fig. 3 Operating characteristic (OC) curve (when AF is unknown).

$\mathrm{Ascov}(\hat{\lambda}_0, \hat{\sigma})$, and $\mathrm{Asvar}(\hat{\sigma})$ are calculated by the MLE of the parameter λ_0, λ_1, and σ.

Let the standardized log censoring time $\zeta_j = (\ln\eta - \lambda_0 - \lambda_1\xi_j)/\sigma, j = 1, 2$, and ζ_j can be expressed using two standardized quantities, a and b:

$$\zeta_j = \frac{\ln\eta - \mu_j}{\sigma} = \frac{\ln\eta - \lambda_0 - \lambda_1\xi_j}{\sigma} = \frac{a - b\xi_j}{\sigma} \tag{9}$$

where $a = (\ln\eta - \lambda_0)/\sigma$, $b = \lambda_1/\sigma$.

The expression of each Fisher information matrix $\boldsymbol{F}_j$ under standardized stress conditions is as follows:

$$\begin{cases} \boldsymbol{F}_j = \frac{1}{\sigma^2}\begin{bmatrix} A_j & \xi_j A_j & B_j \\ & \xi_j^2 A_j & \xi_j B_j \\ \text{symmetric} & & C_j \end{bmatrix} \\ A_j = \Phi(\zeta_j) - \phi(\zeta_j)\{\zeta_j - \phi(\zeta_j)/[1 - \Phi(\zeta_j)]\} \\ B_j = -\phi(\zeta_j)\{1 + \zeta_j(\zeta_j - \phi(\zeta_j)/[1 - \Phi(\zeta_j)])\} \\ C_j = 2\Phi(\zeta_j) - \zeta_j\phi(\zeta_j)\{1 + \zeta_j^2 - \zeta_j\phi(\zeta_j)/[1 - \Phi(\zeta_j)]\} \end{cases} \tag{10}$$

where $\Phi(\cdot)$ is standard normal distribution function and $\phi(\cdot)$ standard normal probability density function.

When $n\rho$ products are tested at a lower stress level, ξ_1 and $n(1-\rho)$ items are tested under higher stress level $\xi_2 = 1$. Fisher information matrix $\boldsymbol{F} = n\rho\boldsymbol{F}_1 + n(1-\rho)\boldsymbol{F}_2$, let $\boldsymbol{H} = \boldsymbol{F}^{-1}/n$ and h_{ij} be the element of matrix $\boldsymbol{H}$. $\mathrm{Asvar}(\hat{\lambda}_0)$, $\mathrm{Ascov}(\hat{\lambda}_0, \hat{\sigma})$, and $\mathrm{Asvar}(\hat{\sigma})$ have a relationship with h_{11}, h_{13}, and h_{33}, respectively, in the matrix $\boldsymbol{H}$. This matrix is the inverse of Fisher information matrix so that the following equation holds true.

$$\mathrm{Asvar}(X) = \frac{\sigma^2}{n}(h_{11} - 2kh_{13} + k^2h_{33}) = \frac{\sigma^2}{n}V^2 \tag{11}$$

where $V^2 = h_{11} - 2kh_{13} + k^2h_{33}$ and V is asymptotic relative standard deviation of test statistic X.

Calculate the matrix $\boldsymbol{H}$ and the following results can be gained:

$$h_{11} = \frac{\rho^2\xi_1^2 D_1 + (1-\rho)^2 D_2 + \rho(1-\rho)(A_1C_2\xi_1^2 - 2B_1B_2\xi_1 + A_2C_1)}{(1-\rho)\rho(1-\xi_1)^2 M},$$

$$h_{13} = \frac{A_1B_2\xi_1 - B_1A_2}{(1-\xi_1)M}, h_{33} = \frac{A_1A_2}{M},$$

$$M = \rho A_2 D_1 + (1-\rho)A_1 D_2, D_j = A_jC_j - D_j$$

2.3.2.3. Operating characteristics (OC) curve. As shown above, the test statistic $X = \hat{\mu} - k\hat{\sigma}$ is asymptotically normally distributed, then the standardized variable U could be written as

$$\begin{aligned} U &= [X - E(X)]/[\mathrm{var}(X)]^{1/2} \\ &= [\hat{\lambda}_0 - k\hat{\sigma} - (\lambda_0 - k\sigma)]n^{1/2}/(\sigma V) \end{aligned} \tag{12}$$

The OC curve $L(p)$ can be written by percent defective p:

$$L(p) = P(X \geqslant Z') = 1 - \Phi(n^{1/2}(u_p + k)/V) \tag{13}$$

where $u_p = Z' - \lambda_0$ is the quantile of the standard normal distribution corresponding to the percent defective p.

To satisfy the requirements of the producer's and consumer's risk, the lots with percent defective $p \leqslant p_\alpha$ are accepted with a probability of at least $1-\alpha$, the lots with $p > p_\beta$ are accepted with a probability of at most β, and the ALSP designed must satisfy the two inequalities:

$$P(X \geqslant Z' | p \leqslant p_\alpha) \geqslant 1 - \alpha \tag{14}$$

$$P(X \geqslant Z' | p \geqslant p_\beta) \leqslant \beta \tag{15}$$

Then the following can be derived:

$$\begin{cases} k^* = \frac{z_{1-\beta}z_{p_\alpha} - z_\alpha z_{p_\beta}}{z_\alpha - z_{1-\beta}} \\ n^* = \left(\frac{z_\alpha - z_{1-\beta}}{z_{p_\alpha} - z_{p_\beta}}\right)^2 V^2 \end{cases} \tag{16}$$

where z_p is the value of quantile p in the standard normal distribution, k^* depends on the two points (p_α, $1-\alpha$) and (p_β, β), and n^* is determined by these two points and V. In order to minimize the sample size n^*, the plan designed should make the value of V minimized.

2.3.2.4. Design of accelerated life testing sampling plan. The theoretical derivation shown in the last section indicates that it would be reasonable to design the testing plans so that V is minimized. The factor V has relationship with parameter ξ_1, ρ and the two standardized quantities a and b where the value of these four parameters is unknown. To obtain the optimal values of ξ_1 and ρ, pre-estimation of a and b is needed. Pre-estimation can be usually obtained from the past experiences, similar test data, and preliminary testing. Here, p_{U} and p_{A} are the probabilities that a product will fail by censoring time η under use and accelerated conditions, respectively. From the expression of $a = (\ln\eta - \lambda_0)/\sigma$ and $b = \lambda_1/\sigma$, the parameters a and b can be expressed by p_{U} and p_{A} as follows:

$$\begin{cases} a = \Phi^{-1}(p_{\mathrm{U}}) \\ b = \Phi^{-1}(p_{\mathrm{U}}) - \Phi^{-1}(p_{\mathrm{A}}) \end{cases} \tag{17}$$

V can be minimized using pre-estimation of p_{U} and p_{A}, and then the optimization results of ξ_1 and ρ can be obtained. The optimal values of ξ_1^* and ρ^* can be obtained by getting the minimum V^* using a numerical search method with computer program, then bringing V^* to Eq. (16), we can obtain the optimal value of sample size n^*. Table 2 shows optimum time-censored ALTSPs for selected combinations of two points of OC curve and (p_{U}, p_{A}).

Table 2 ALSPs under time-censoring, $p_\alpha = 0.0002$, $p_\beta = 0.03$; $(\alpha, \beta) = (0.2, 0.2)$.

p_{U}	p_{A}	ξ^*	ρ^*	n^*
0.000006	0.999	0.44	0.77	13
	0.632	0.58	0.75	45
	0.332	0.62	0.73	91
0.000045	0.999	0.40	0.78	11
	0.632	0.53	0.78	35
	0.332	0.56	0.76	66
0.000335	0.999	0.35	0.79	10
	0.632	0.46	0.81	25
	0.332	0.47	0.81	43
0.002476	0.999	0.28	0.78	8
	0.632	0.34	0.86	16
	0.332	0.32	0.88	23

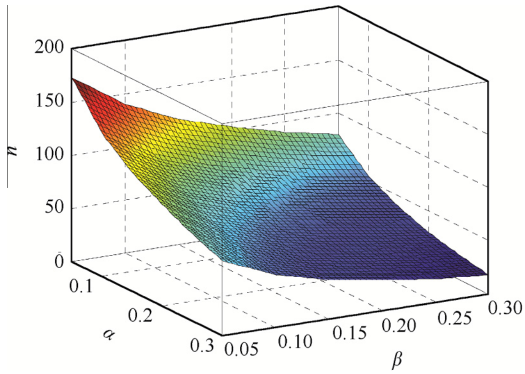

Fig. 4 Sample size change with α and β (when AF is unknown).

Some of the characteristics of this plan are shown in Table 2:

(1) When (p_U, p_A), p_α, p_β are given, and changes in either risk only affect the size of sample n, then the values of ξ_1 and ρ, and n decrease as risks increase. This is because the optimization of ξ_1 and ρ does not depend on the choice of both risks.

(2) When two points (p_α, $1-\alpha$) and (p_β, β) of OC curve and p_A are given, as p_U increases, sample size decreases. When two points and p_U are given, as p_A increases, sample allocation ratio ρ increases and the low level stress s_1 decreases. Sample size and the sample allocation ratio decrease and low level stress s_1 increases, and as p_U decreases, this trend has become less obvious.

(3) Under the condition of p_U (p_A) being unchanged, the product has better acceleration when p_A (p_U) is greater, and the sample needed is less.

Similarly, to explore how sampling plan changes with α and β, set $p_\alpha = 0.0002$, $p_\beta = 0.03$; $p_U = 0.000006$, $p_A = 0.632$, respectively. α and β were changed to determine their influences on the sample size n, as shown in Fig. 4.

3. Case study

3.1. Known AF

3.1.1. ALSP design

As experienced by engineers under real-world conditions, the life of an electronic product partly follows lognormal distribution. For a certain type of electronic component, the life has acceleration to the electric stress, stress and some life characteristics of the electronic products obey the inverse power law model.

Operating stress V_U is 15 kV, and tests are conducted under accelerated stress conditions $V_A = 36$ kV. According to empirical estimates, AF = 14.

Suppose that the censoring time at the accelerated condition $\tau_A = 1000$ h, $\mu_{U0} = 100000$ h, $\mu_{U1} = 50000$ h, $\sigma = 2500$, $\alpha = 0.2$ and $\beta = 0.2$. Using Eq. (6), q_0 and q_1 can be calculated:

$$q_0 = 1 - \Phi\left(\frac{\ln 1000 - 100000/14}{2500}\right) = 0.994$$

$$q_1 = 1 - \Phi\left(\frac{\ln 1000 - 50000/14}{2500}\right) = 0.859$$

Then, as shown in Table 1, the related sampling test plans can be developed. When $q_0 = 0.99$ and $q_1 = 0.86$, the plan (n, c) is (21, 2). In this way, a sampling plan can be implemented as follows: 21 products are randomly selected from a lot and tested under accelerated conditions with 36 kV. The accelerated life test is terminated at the censoring time 1000 h. If it involves 2 failures or more, the lot is rejected. Otherwise, it is accepted.

3.1.2. Analysis of sensitivity to AF

Here, the accelerated method is used in the test plan, so the influence of the changes in AF on test plans must be considered. This section discusses the degree of sensitivity of test plan for changing of AF. Let AF and AF* be the assumed and true acceleration factors, where, from Eq. (6), the true reliability of product at the censoring time τ_A under hypotheses can be written as

$$q_i^* = 1 - \Phi\left(\frac{\ln \tau_A - \mu_{Ui}/\mathrm{AF}^*}{\sigma}\right)$$

The uncertainty of AF is usually described as a range, as follows:

$$\mathrm{AF}^* \in \left[\mathrm{AF}^*_{\min}, \mathrm{AF}^*_{\max}\right] \tag{18}$$

When both risks are determined, the sample size and rejection number can be adjusted by calculating the assumed value and true value in the range of the uncertainty of AF.

Now, design a related sampling plan considering the uncertainty of AF. In the case above, suppose the change range of the AF is $\pm 5\%$. The relevant q_0^* and q_1^* can be calculated as follows when $\mathrm{AF}^*_{\max} = 14(1 + 5\%)$:

$$q_0^* = 1 - \Phi\left(\frac{\ln 1000 - 100000/14 \times (1+5\%)}{2500}\right) = 0.990$$

$$q_1^* = 1 - \Phi\left(\frac{\ln 1000 - 50000/14 \times (1+5\%)}{2500}\right) = 0.841$$

Again, take $\alpha = 0.2$ and $\beta = 0.2$ as an example, the sampling plan $(n, c) = (18, 2)$ is determined. When $\mathrm{AF}^*_{\min} = 14(1-5\%)$, the following is true:

$$q_0^* = 1 - \Phi\left(\frac{\ln 1000 - 100000/14 \times (1-5\%)}{2500}\right) = 0.996$$

$$q_1^* = 1 - \Phi\left(\frac{\ln 1000 - 50000/14 \times (1-5\%)}{2500}\right) = 0.871$$

The relevant sampling plan $(n, c) = (24, 2)$ is determined. When AF varies as the value $\mathrm{AF}_0 = 14$ in the range [−15%, 15%] Fig. 5 shows how the relative change of sample size Δn changes with the increase of α and β. Herein, $\Delta n = (n - n^*)/n^*$ and n^* is the sample size of the selected plan. Some exact results are listed in Table 3. From Fig. 5 and Table 3, the following is true:

(1) A small change of AF in the range can make the test plan change, the bigger the change in AF, the greater the change in the test plan.

(2) When both risks are low, the impact of AF on the test plan becomes more significant.

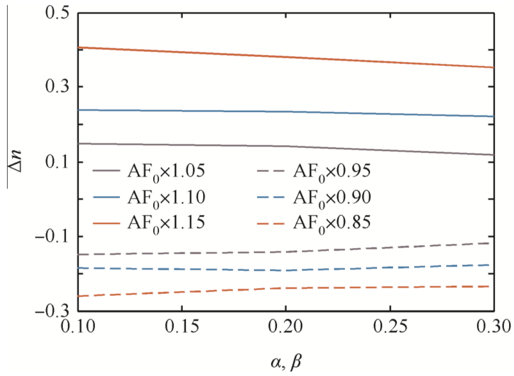

Fig. 5 Analysis of sensitivity to AF.
Note: x-axis means that producer and customer risks take the same value at the same time, e.g., $\alpha = \beta = 0.2$, etc.

Table 3 Analysis of sensitivity to AF.

α, β	α^*	β^*	Selected plan (n^*, c)	Relative change in AF (%)	After the change (n, c)	Δn (%)
0.1	0.08	0.098	(27,2)	+5	(23,2)	−14.81
0.1	0.04	0.099	(27,2)	−5	(31,2)	+14.81
0.2	0.15	0.192	(21,2)	+5	(18,2)	−14.29
0.2	0.13	0.198	(21,2)	−5	(24,2)	+14.29
0.3	0.24	0.282	(17,2)	+5	(15,2)	−11.76
0.3	0.22	0.289	(17,2)	−5	(19,2)	+11.76

Note: $\alpha^*$and β^* are the actual risks.

An analysis of sensitivity to AF is shown in Fig. 5. Hence, before designing the plan, we should analyze and estimate the range of AF and decide both risks according to the actual situation.

3.2. Unknown AF

3.2.1. ALSP design

In the case above, the AF is unknown. Under use conditions, the lower limit of this product's lifetime is 50000 h. The voltage under use conditions is 15 kV. The time-censored ALTSP is then designed as follows:

Step 1. Choose two points (p_α, $1-\alpha$) and (p_β, β) on the OC curve and pre-estimate parameters (p_U, p_A). Let two points be (p_α, $1-\alpha$) = (0.0002, 0.8), (p_β, β) = (0.03, 0.2), and (p_U, p_A) = (0.000335, 0.999). Let high stress level be 39 kV. The log transformed stresses are s_0 = ln(15,000) = 9.6158 and s_2 = 10.5713.
Step 2. Calculate k^* from Eq. (16), since $z_\alpha = -1.645$, $z_{1-\beta} = 1.282$, $z_{p_\alpha} = -3.54$, and $z_{p_\beta} = -1.881$. This yields $k^* = 2.7104$.
Step 3. Determine ξ_1^* and ρ^* by obtaining the minimum V^*, from Table 2. This yields $\xi_1^* = 0.35$, $\rho^* = 0.79$, where $V^* = 3.0535$. Since $\xi_1^* = 0.35$, the low stress level can be found: s_1 = 21 kV.
Step 4. Compute n^* using Eq. (16). This yields $n^* = 10$.

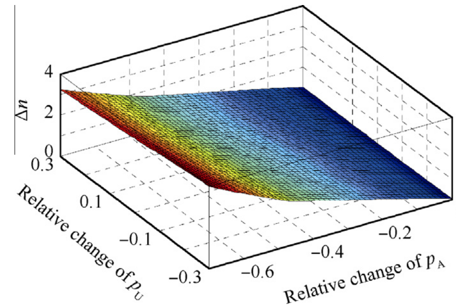

Fig. 6 Sample size change with p_U *and* p_A.

Step 5. Because $\rho^* = 0.79$ and $n^* = 10$, the sample size allocated to the low stress s_1 is 8 and that allocated to the high stress s_2 is 2.

The time-censored ALSP can be implemented as follows: Ten products are randomly selected for use as samples. Among these, 8 products are allocated to the low stress 21 kV and 2 products test at the high stress 39 kV. The test is run until the censoring time g is reached. Using the data from the life test, the MLE $\hat{\mu}_U$of parameter μ under use conditions is estimated. If test statistic X is greater than ln(50000) = 10.82, then the lot is accepted. Otherwise, it is rejected.

3.2.2. Analysis of sensitivity to p_U and p_A

To design the test plan for use when AF is unknown, the probabilities p_U and p_A that a product will fail by censoring time η under the use and accelerated conditions must be known in advance. However, these settings are usually not accurate, so how p_U and p_A affect testing plan will be studied in this section. A three-dimensional plot of Δn as a function of relative change of p_U and p_A is shown in Fig. 6. When parameter p_U is varied as the value p_U = 0.000335 in the range [−30%, 30%] and p_A = 0.999 in the range [−70%, 0%] (the other parameter settings keep the same), the largest Δn = 380%. We can also realize that changes in p_U have less impact on the test plan than changes in p_A. It allows the user to assess differences in the samples. This shows that differences in the acceleration of the product are the main factors that influence the plan.

4. Conclusion and future work

This paper describes the design of accelerated life testing sampling plans for lognormal lifetime distribution under time-censoring condition. Take both producer and consumer risks into account. Then, two ALSPs are designed, one for known-AF conditions and one for unknown-AF conditions. The following conclusions are reached:

(1) When AF is known, the ALT sampling plan has the following properties:
   a) When both risks are given, the sample size and rejection number increase as the reliability of the lower limit increases and decreases as the reliability of the upper limit increases.

b) When the upper and lower limits are given, sample size and rejection number decrease as both risks increase.
c) The impact of changes in AF on the testing plan decreases as both risks increase.

(2) When AF is unknown, the plan has the following properties:
a) Both risks and the inherent acceleration of products can influence the choice of sample size and stress level. When both risks increase, the sample size decreases. When the acceleration of the product is stronger, then a smaller validation sample could be used.
b) Changes in p_U have less impact on the test plan than changes in p_A.

When AF is known, through the sensitivity analysis of the AF, the fact that the fluctuation of AF has more influence on ALSPs than on other factors can be derived. The product's prior information is needed. When AF is unknown, some parameters must be pre-estimated using prior information. Bayes theory can be used to consider prior information during the design of ALSPs. An appropriate model selection procedure (such as one based on BIC, AIC criteria, or MSE) should be used to choose a suitable acceleration function.

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